Relations Among Ballistic Properties of Solid Propellants

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Nomenclature

a = rate coefficient
A_b = burning surface area
A^* = nozzle throat area
C^* = characteristic velocity
g_c = unitary constant
K = area ratio, A_b/A^*
L^* = characteristic length, chamber volume/throat area
M = molecular weight
n = pressure exponent, $(\partial \ell_n r_0/\partial \ell_n p)_{T_i}$
n_f = generalized pressure sensitivity, $(\partial \ell_n f/\partial \ell_n p)_{T_i}$
p = pressure
r = burning rate
r_0 = burning rate in quiescent environment
t = time
T_f = flame temperature
T_i = propellant temperature
x = distance along port
α = propellant's volume expansivity, $-(\partial \ell_n \rho_c/\partial T_i)_p$
β_T = propellant's compressibility, $(\partial \ell_n \rho_c/\partial p)_{T_i}$
$\pi_{f,g}$ = generalized temperature sensitivity, $(\partial \ell_n f/\partial T_i)_g$
ρ_c = propellant density
ρ_f = gaseous products density
ρ_R = density ratio, ρ_f/ρ_c
σ_f = generalized burning rate temperature sensitivity, $(\partial \ell_n r_0/\partial T_i)_f$

Introduction

THE temperature sensitivity of solid propellants has been and is of considerable interest because of its importance to both performance [1-3] and stability.[4-6] A recent study by Cohen and Flanigan[7,8] reported data that demonstrated the dependence of temperature (and pressure) sensitivity upon the pressure and propellant temperature. Because these sensitivities are obtained experimentally from both strand and ballistic test motor data, the relations connecting these two *environments* are useful. Relations of this type have been derived for constant-pressure exponent.[9] However, as the rate sensitivities may not be constants, Cohen and Flanigan derived a new expression relating the motor pressure and burning rate temperature sensitivities. Unfortunately, their result is incomplete (density changes were neglected) and confusing (a second-order term appeared in the denominator). Because the temperature sensitivity is small $[\mathcal{O}(10^{-2} - 10^{-3})\mathrm{K}^{-1}]$ and must be computed from absolute rate measurements (small-difference measurement techniques are unavailable), accurate and consistant burning rate data are crucial to obtaining reasonably accurate sensitivities. To this end Geckler and Sprenger[10] derived relations for testing the consistency of ballistic test motor data from semiempirical relations by assuming constant coefficients. Because these relations could be employed to improve the quality of the rate data derived from ballistic test motors by identifying questionable results, they offered improved accuracy if they were valid for the variable-sensitivity situation identified by Cohen and Flanigan.[7] The objective of this work is to present a unified and general development of relations among ballistic properties.

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Analysis

With usable solid propellants in a quiescent environment, the dependence of any ballistic property f is a well-behaved function of pressure and propellant temperature $[f=f(p,T_i)]$. Therefore, the differential

$$\mathrm{d}\ell_n f = n_f \mathrm{d}\ell_n p + \pi_{f,p}\mathrm{d}T_i \tag{1}$$

is exact and the test for exactness[11] gives the Maxwell relations

$$\left(\frac{\partial n_f}{\partial T_i}\right)_p = \left(\frac{\partial \pi_{f,p}}{\partial \ell_n p}\right)_{T_i} \tag{2}$$

where $n_f = (\partial \ell_n f/\partial \ell_n p)_{T_i}$ and $\pi_{f,g} = (\partial \ell_n f/\partial T_i)_g$ are generalized pressure and temperature sensitivities, respectivity. The most important of these occurs when $f=r_0$ which yields

$$\mathrm{d}\ell_n r_0 = n\mathrm{d}\ell_n p + \sigma_p \mathrm{d}T_i \tag{3}$$

and

$$\left(\frac{\partial n}{\partial T_i}\right)_p = \left(\frac{\partial \sigma_p}{\partial \ell_n p}\right)_{T_i} \tag{4}$$

Consequently, the temperature dependence of the pressure exponent leads to the pressure dependence of the temperature sensitivity of the burning rate. This situation is alluded to by Brooks and Miller,[9] but not demonstrated.

For any well-behaved propellant function $f(p,T_i)$ in a burner with a constant-area ratio K, it can be shown that

$$\pi_{f,K} = \pi_{f,p} + n_f \pi_{p,K} \tag{5}$$

With $f = r_0$, ρ_c, and C^*, Eq. (5) becomes respectively

$$\sigma_K = \sigma_p + n\pi_{p,K} \tag{6}$$

$$\pi_{\rho_c,K} = \pi_{\rho_c,p} + n_{\rho_c}\pi_{p,K} \tag{7}$$

$$\pi_{C^*,K} = \pi_{C^*,p} + n_{C^*}\pi_{p,K} \tag{8}$$

As volume expansivity is $\alpha = -(\partial \ell_n \rho_c/\partial T_i)_p = -\pi_{\rho_c,p}$ and the isothermal compressibility is $\beta_T = (\partial \ell_n \rho_c/\partial p)_{T_i}$, $n_{\rho_c} = p\beta_T$ and

$$\pi_{\rho_c,K} = -\alpha + p\beta_T\pi_{p,K} \tag{9}$$

In well-designed ballistic test motors, the pressure-time history is neutral, the pressure is essentially invariant along the port, and the erosive burning is negligible. Therefore, $r(x,t) \approx r_0[p(t)]$ and the bulk-mode quasisteady conserva-

Table 1 Consistency relations for ballistic test motors

Geckler and Sprenger[10]	This work
$\pi_{p,K} = \sigma_K + \pi_{C^*,K}$	$\pi_{p,K} = \sigma_K + \pi_{C^*,K} + \pi_{\rho_c,K}$
$\pi_{C^*,p} = \pi_{C^*,K} - n_{C^*}\pi_{p,K}$	$\pi_{C^*,p} = \pi_{C^*,K} - n_{C^*}\pi_{p,K}$
$\sigma_p = \sigma_K - n\pi_{p,K}$	$\sigma_p = \sigma_K - n\pi_{p,K}$
$n_K = -n_{C^*} - n + 1$	$n_K = -n_{C^*} - n + 1$
$\pi_{K,p} = -\pi_{C^*,p} - \sigma_p$	$\pi_{K,p} = \alpha - \pi_{C^*,p} - \sigma_p$
$\pi_{p,K} = -\pi_{K,p}/n_K$	$\pi_{p,K} = -\pi_{K,p}/n_K$

tion of mass is an excellent approximation. Consequently,

$$\ell n r_0 + \ell n \rho_c + \ell n K + \ell n (1 - \rho_R) = \ell n p + \ell n g_c - \ell n C^* \tag{10}$$

where the $1-\rho_R$ term accounts for differences between the gas- and condensed-phase densities. Differentiating with respect to T_i while holding p constant and assuming the fluid is a perfect gas gives

$$\sigma_p + \pi_{K,p} + \pi_{C^*,p} - \frac{\alpha}{(1-\rho_R)} + \frac{\rho_R \pi_{T_f,p}}{(1-\rho_R)} = 0 \tag{11}$$

Differentiating with respect to $\ell n p$ while holding T_i constant yields

$$n + n_K + n_{C^*} + [p\beta_T + \rho_R (n_{T_f} - 1)] / (1 - \rho_R) = 1 \tag{12}$$

Finally, differentiating with respect to T_i while holding K constant gives

$$\sigma_K + \pi_{C^*,K} + (\pi_{\rho_c,K} + \rho_R \pi_{T_f,K} - \pi_{p,K}) / (1-\rho_R) = 0 \tag{13}$$

With Eqs. (6), (8), and (9), Eq. (13) becomes

$$\pi_{p,K} = \frac{\sigma_p + \pi_{C^*,p} + (\rho_R \pi_{T_f,p} - \alpha)/(1-\rho_R)}{1 - n - n_{C^*} - [p\beta_T - \rho_R (1 - n_{T_f})]/(1-\rho_R)} \tag{14}$$

In ballistic test motors, $\bar{p} = \bar{p}(K, T_i)$. Therefore, with the cyclic relation for $f(x,y,z) = 0$, $(\partial x/\partial y)_z (\partial y/\partial z)_x (\partial z/\partial x)_y = -1$,

$$\pi_{p,K} = -\pi_{K,p}/n_K \tag{15}$$

Discussion

In solid rocket motors $\mathcal{O}(p) \sim 10$ MPa, $\mathcal{O}(M) \sim 20$, and $\mathcal{O}(T_f) \sim 3000$ K. When $\mathcal{O}(\rho_c) \sim 1.8$ g/cm^{-3}, $\mathcal{O}(\rho_R) \sim 5 \times 10^{-3}$. Therefore, as $\mathcal{O}(\pi_{T_f,p}) \sim 10^{-3}$K^{-1}, $\mathcal{O}(\beta_T) \sim 10^{-4}$ MPa^{-1} (see Ref. 12), $\mathcal{O}(\pi_{\rho_c,K}) \sim \pi_{\rho_c,p} \sim -\alpha$, and $\mathcal{O}(\alpha) \sim 3 \times 10^{-4}$K^{-1} (see Ref. 12), Eqs. (11-13) can be simplified with an error less than 1% to

$$\sigma_p + \pi_{K,p} + \pi_{C^*,p} \approx \alpha \tag{16}$$

$$n + n_K + n_{C^*} \approx 1 \tag{17}$$

$$\sigma_K + \pi_{C^*,K} - \pi_{p,K} \approx -\pi_{\rho_c,K} \tag{18}$$

In addition, as the denominator of Eq. (14) is $\mathcal{O}(0.1)$, it can be simplified to

$$\pi_{p,K} \approx (\sigma_p + \pi_{C^*,p} - \alpha)/(1-n) \tag{19}$$

Equation (19) shows that, with the addition of a volume expansivity term (apparently first noted by Fling[13]), the relation between the motor pressure and rate temperature sensitivities is the same for both constant and variable sensitivities.

Table 1 summarizes the consistency relations derived by Geckler and Sprenger[10] (in the nomenclature of this work) and compares them with those derived herein [Eqs. (6), (8), (15-18)]. With the exception of the density-related terms neglected by Geckler and Sprenger[10] and those terms that can be neglected with only a small error in most typical situations, they are identical. The relations derived herein are general, as is the Geckler and Sprenger procedure.

Conclusions

A Maxwell relation exists between pressure and temperature rate sensitivities. The methodology developed by Geckler and Sprenger for checking the consistency of data obtained from ballistic test motors is general. Relations connecting motor pressure and rate sensitivities to propellant temperatures are identical for either constant or variable burning rate parameters.

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